



Secret of the Talent Show

Does the order of the contestants' performance in a talent show effect the probability of them winning?



At present the three biggest talent shows on UK television are:

- Strictly Come Dancing (Strictly) - a contest between celebrities who ballroom dance with a professional dancer;
- The X Factor (X Factor) - a talent show for singers/groups;
- Britain's Got Talent (BGT) - a talent show for all types of performers.

In Strictly and X Factor contestants perform in a certain order. The public vote by phone or text and the two contestants with the lowest votes have to sing or dance again for the judges to decide who is eliminated. This **Britain's Got Talent (BGT)**

There have been six series of BGT which first began in 2007. Between 2007 and 2011 there were eight acts in each semi final. The contestants performed and the act with the highest public vote automatically went through to the final. The judges then decided whether the runner-up or second runner-up would go through.

So,

Does the order of the contestants' performance in BGT affect the probability of them winning?

TASK A

Let's consider BGT Series 1.

There were 3 BGT semi-finals in Series 1. The results are on the next page.



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 Winner
 Runner up

Semi-final 1

Order	Finished	Artist	Act
1	4th	MD Productions	Dance Group
2	2nd (won judges' vote)	Damon Scott	Miming Puppet Act
3	8th	Mel's Clever K9'S	Line Dancing Dog Act
4	3rd (lost judges' vote)	Dominic Smith	Singer
5	6th	Luke & Charlotte	Dancing Couple
6	7th	Caroline Boyes	Solo Dancer
7	5th	The Free Runners	Aerobics Act
8	1st (won public vote)	Paul Potts	Opera singer

Semi-final 2

Order	Finished	Artist	Act
1	2nd (won judges' vote)	Kombat Breakers	Dance group
2	5th	Victoria Armstrong	Angle Grinding
3	6th	Jack Reeve	Tap Dancer
4	7th	Jake Pratt	Singer/Comedian
5	8th	The Mini-Mezzos	Dance Group
6	4th	Crazeehorse	Acrobatic Act
7	3rd (lost judges' vote)	Craig Womersley	Baton Twirler
8	1st (won public vote)	Bessie Cursons	Musical theatre performer

Semi-final 3

Order	Finished	Artist	Act
1	7th	Cheeky Bits	Dance Group
2	4th	Mike Garbutt	Comedian
3	8th	Doctor Gore	Alternative Magic/Comedy
4	3rd (lost judges' vote)	Tony Laf	Singer
5	6th	Scott Holtom	Solo Dancer
6	5th	Crew 82	Beat boxing group
7	2nd (won judges' vote)	The Bar Wizards	Alternative Juggling
8	1st (won public vote)	Connie Talbot	Singer

You will notice in each case the winner of the semi-final performed last.

Is this likely to have occurred at random?

Let's make some assumptions so we can calculate the probability of this happening.

Assumption 1

Each contestant has the same probability of winning.

So, the probability of each performer winning is $\frac{1}{8}$ as there are 8 contestants.

Does this seem reasonable? Give a reason for your answer.



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Assumption 2

The probability of a single contestant winning and the probability of them losing sum to 1, since there are only 2 outcomes win or lose.

Does this seem reasonable? Give a reason for your answer.

Assumption 3

The result of each semi-final is independent, i.e. the result of 1 semi-final does not affect the result of another.

Does this seem reasonable? Give a reason for your answer.

Assumption 4

The probability of the last performer winning is equal for each semi-final.

Does this seem reasonable? Give a reason for your answer.

If we consider these 4 assumptions to be true then the probability of the last performer winning each of the 3 semi-finals is

$$\left(\frac{1}{8}\right)^3 = 0.0020 \text{ 2 sf}$$

Therefore, very unlikely.

Note: Any particular result for these three semi-finals also has a probability of 0.0020 of happening. For example if you wanted the 1st act to win in Semi-final 1, the 3rd act to win in Semi-final 2 and the 4th act to win in Semi-final 3 this would also have a probability of

$$\left(\frac{1}{8}\right)^3 = 0.0020 \text{ 2 sf}$$



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TASK B

Consider BGT Series 2. There were 5 semi-finals in Series 2.

Semi-final 1

Order	Finished	Artist	Act
1	6th	Boogie Wonderland	16-strong dance group
2	2nd (won judges' vote)	Kate and Gin	Musical canine freestyle
3	8th	Michael Machell	Keyboardist
4	3rd (lost judges' vote)	Dean Wilson	Musical theatre singer
5	5th	Sophie Mei	Belly-dancing
6	4th	Tracey Lee Collins	Singer
7	7th	Phil Blackmore ^[note 1]	Balancing/alternative juggling
8	1st (won public vote)	Signature	Michael Jackson/Bhangra dance act

Semi-final 2

Order	Finished	Artist	Act
1	5th	Mandy Ellen Dancers	39-strong dance group
2	4th	Iona	Contortionist
3	2nd (won judges' vote)	Cheeky Monkeys	Junior dance couples
4	8th	Kay and Harvey	Opera and electronic keyboard
5	7th	Bang On!	Urban percussionists
6	3rd (lost judges' vote)	Flava	Hip-hop dance group
7	6th	Sauris Nandi	Deceptional and illusionary magic
8	1st (won public vote)	Andrew Johnston	Boy soprano

Semi-final 3

Order	Finished	Artist	Act
1	4th	Irresistible	Pop group
2	2nd (won judges' vote)	Strike	Martial arts demo
3	8th	Mary Halford March	24-strong dance group
4	7th	The Deans Of Magic	'Erotic' magic
5	3rd (lost judges' vote)	Charlie Green	Singer
6	6th	Urban Gypsies	7-strong belly-dancing group
7	5th	Hoop La La	Hula hoop entertainment
8	1st (won public vote)	George Sampson	Breakdancing

Semi-final 4

Order	Finished	Artist	Act
1	4th	James Stone	Pop singer
2	5th	Charlie Wernham	Stand-up comedian
3	8th	Harlequin Stage School	22-strong dance group
4	2nd (won judges' vote)	Andrew Muir	Pop singer
5	6th	The Boogie Babes	19-strong dance group
6	7th	Vizage ^[note 2]	Quick-change act
7	3rd (lost judges' vote)	Jeremy Lynch	Football player
8	1st (won public vote)	Faryl Smith	Classical singer

Semi-final 5

Order	Finished	Artist	Act
1	8th	Caburlesque	Cabaret/Burlesque
2	4th	Madonna Decena	Pop singer
3	5th	Anya Sparks	Dancer
4	3rd (lost judges' vote)	Craig Harper	Pop/impersonist
5	7th	Diva Las Vegas	7-strong cabaret act
6	2nd (won judges' vote)	Nemesis	5-strong troupe
7	6th	Per Diem	Guitar and vocals
8	1st (won public vote)	Escala	Electric string quartet

If we consider the 4 assumptions in TASK A to be true calculate the probability of the last performer of each of the 5 semi-finals wins.



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TASK C

Consider BGT Series 3. There were 5 semi-finals in Series 3.

Semi-final 1

Order	Finished	Artist	Act
1	2nd (36.2%)	Diversity	Street dancers
2	5th	Sue Son	Electric violinist
3	8th	Darth Jackson	Darth Vader/Michael Jackson impersonator
4	3rd	Natalie Okri	Singer
5	6th	Julia Naidenko	Belly Dancer
6	7th	Nick Hell	Freakshow/stunt performer
7	4th	Faces of Disco	Comedy dancers
8	1st (52.1%)	Susan Boyle	Singer

Semi-final 2

Order	Finished	Artist	Act
1	5th	DJ Talent	Rapper
2	4th	Merlin Cadogan	Escapologist
3	8th	Hot Honeyz	Dancers
4	6th	Jamie Pugh	Singer
5	7th	Peter Coghlan (aka Mama Trish)	Drag act
6	3rd (lost judges' vote)	Gareth Oliver	Ventriloquist
7	2nd (37.1%)	Shaun Smith	Singer
8	1st (38.7%)	Flawless	Street dance troupe

Semi-final 3

Order	Finished	Artist	Act
1	4th	Harmony	Musical theatre performers
2	7th	Kay Oresanya	Human saxophone
3	6th	Ben and Becky	Balroom dancers
4	2nd (26.3%)	Shaheen Jafarhooi	Singer
5	5th	The Barrow Boys	Wheelbarrow dancers
6	3rd (lost judges' vote)	MD Showgroup	Dancers
7	8th	Floral High Notes	Opera singing and flower arranging
8	1st (50.4%)	Stavros Flatley	Comedy Dancers

Semi-final 4

Order	Finished	Artist	Act
1	8th	Sugar Free	Dancers
2	7th	Jackie Prescott and Tippy Toes	Dog act
3	3rd (lost judges' vote)	Callum Francis	Musical performer
4	5th	Fred Bowers	Break dancer
5	6th	Brit Chix	Rock band
6	1st (56.7%)	Julian Smith	Saxophonist
7	2nd (24.6%)	2 Grand	Singers
8	4th	Fabia Cerra	Burlesque dancer

Semi-final 5

Order	Finished	Artist	Act
1	7th	The Dreambears	Comedy dancers
2	6th	Good Evans	Singers
3	8th	Luke Clements	Juggler
4 & 7	2nd (26.0%)	Hollie Steel	Singer
5	5th	Martin Matcham	Singer and guitarist
6	1st (50.6%)	Aidan Davis	Dancer
8	4th	DCD Seniors	Dancers
9	3rd (lost judges' vote)	Greg Pritchard	Countertenor singer

What is different about the results for these 5 semi-finals?



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So now we have 3 of the acts who performed last winning out of 5 shows.

To find the probability of this happening we need some extra notation.

Consider a BGT semi-final with 2 shows, each with 8 contestants.
How many ways can the last person to perform win 1 show only?

W = Last person to perform wins L = Last person to perform loses

	Show	
	1	2
Combination 1	W	L
Combination 2	L	W

This can happen in 2 ways. So the probability that the last person to perform will win the show is:

This can happen in 2 ways

$$2 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{7}{8}\right)^1 = 0.22 \text{ (2 sf)}$$

Probability last person to perform wins 1 show
Probability last person to perform loses 1 show

Now consider 3 semi-finals with 8 contestants. How many ways can the last person to perform win 1 show?

	Show		
	1	2	3
Combination 1	W	L	L
Combination 2	L	W	L
Combination 3	L	L	W

This can happen in 3 ways.



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There is a quicker way to work this out using

$${}^n C_r = \frac{n!}{(n-r)! r!}$$

$${}^3 C_1 = \frac{3!}{(3-1)! 1!}$$

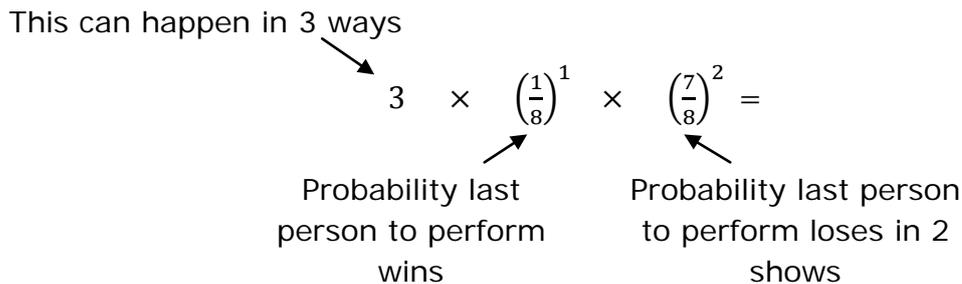
$${}^3 C_1 = \frac{3!}{2! 1!}$$

$${}^3 C_1 = \frac{3 \times 2 \times 1}{2 \times 1 \times 1}$$

$$= \frac{3 \times 2^1 \times 1}{2^1 \times 1 \times 1} = 3$$

Note: $n! = n \times (n-1) \times (n-2) \times \dots \times 1$
 For example: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

So the probability that the last person to perform will win in only one show is:



Consider a BGT semi-final with 4 shows, each with 8 contestants.
 How many ways can the last person to perform win in only 2 of the shows?

$${}^n C_r = \frac{n!}{(n-r)! r!}$$

$${}^4 C_2 = \frac{4!}{(4-2)! 2!}$$

$${}^4 C_2 = \frac{4!}{2! 2!}$$

$${}^4 C_2 = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$$

$$= \frac{4^2 \times 3 \times 2 \times 1}{2^1 \times 1 \times 2^1 \times 1} = 6$$

Show	1	2	3	4
Combination 1	W	W	L	L
Combination 2	W	L	W	L
Combination 3	W	L	L	W
Combination 4	L	W	W	L
Combination 5	L	W	L	W
Combination 6	L	L	W	W

There are 6 ways this can happen.



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So the probability that the last person to perform will win 2 shows out of 4 is:

This can happen in 6 ways

$$6 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{7}{8}\right)^2 = 0.072 \text{ (2 sf)}$$

Probability last person to perform wins 2 shows

Probability last person to perform loses in 2 shows

What is the probability that, in a BGT semi-final with 5 shows each with 8 contestants, the last person to perform will win in only 3 shows?

The Binomial Distribution.

The binomial distribution represents: the number of successes (r) in a fixed number of trials (n).

Assumptions:

p is the probability of success, q is the probability of failure.

$p + q = 1$.

The trials are independent.

The probability of success is the same for each trial.

If $X \sim \text{Bin}(n, p)$, then

$$P(X = r) = {}^n C_r p^r q^{n-r} \text{ for } 0 \leq r \leq n$$

where X is the number of times the last act to perform wins in 5 BGT semi-finals.

The BGT model above is an example of the binomial distribution, if we accept the assumptions in TASK A. However, assumptions 1 and 4 may not hold as the contestants may not have equal talent.



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TASK D

Consider BGT Series 5. There were 5 semi-finals in Series 5.

Semi-final 1 (30 May)

Order	Finished ⁽²⁾	Artist	Act
1	7th - 1.0%	Girls Roc	Dancing Fire Act
2	5th - 4.0%	Donelda Guy	Dog Act
3	8th - 0.4%	Stuart Arnold	Terminator Impersonator/Rapper
4	2nd - 15.1%	Paul Gbegbaje	Pianist
5	4th - 8.2%	Bruce Sistaz	Martial Artists
6	6th - 2.5%	Ted & Grace	Grandfather & Granddaughter Singing Duo
7	3rd - 9.2%	Joe Oakley	Trial Rider
8	1st - 61.6%	Ronan Parke	Singer

Semi-final 2 (31 May)

Order	Finished ⁽²⁾	Artist	Act
1	4th - 15.4%	Two and a Half Men	Dance Trio
2	7th - 1.8%	Enchantment	Acrobatic Circus Troupe
3	5th - 5.2%	Herbie Armstrong	Singer/Guitarist
4	8th - 1.5%	Lorna Bliss	Britney Spears Impersonator
5	1st - 34.3%	New Bounce	Boyband
6	2nd - 20.8%	Jean Martyn	Organist
7	3rd - 16.5%	David & Karen	Illusionists
8	6th - 4.5%	Up & Over it	Table Hand Dancers

Semi-final 3 (1 June)

Order	Finished ⁽²⁾	Artist	Act
1	6th - 4.0%	The Circus of Horrors	Extreme Variety Show
2	4th - 7.8%	Jay Worley	Singer
3	7th - 1.2%	Angela & Teddy	Dog Tricks
4	5th - 4.5%	Abyss	Street Dancers
5	8th - 0.7%	Wachirapom Tirpak	Entertainer
6	1st - 38.1%	Les Gibson	Impressionist
7	2nd - 33.0%	James Hobbly	Solo Dancer
8	3rd - 10.7%	Gay and Alan	Handbell Players

Semi-final 4 (2 June)

Order	Finished ⁽²⁾	Artist	Act
1	6th - 2.3%	Dance Angels Elite	Dance Troupe
2	5th - 4.7%	Jessica Hobson	Singer/Pianist
3	8th - 0.4%	Mexican Mayhem	Trained Chihuahuas
4	4th - 11.9%	Out of the Blue	Acapella Group
5	7th - 1.0%	Nathan Wyburn	Visual Artist
6	1st - 42.9%	Jai McDowall	Singer
7	3rd - 13%	Steven Hall	Comedic Dancer
8	2nd - 23.9%	Edward Reid	Comedic Singer

Semi-final 5 (3 June)

Order	Finished ⁽²⁾	Artist	Act
1	7th - 1.1%	Marawa	Hula Hoop Artist
2	3rd - 12.0%	Pip & Puppy	Opera Singer & Singing Dog
3	4th - 8.1%	The Celtic Colleens	Blacklight Irish Dancers
4	9th - 0.3%	Mr. & Mrs.	Singers/Pianist
5	6th - 5.6%	Follow the Right Path	Rappers
6	8th - 0.6%	Antonio Popeye ⁽³⁾	Eye-Popper
7	5th - 6.8%	Michael Moral ⁽¹⁾	Breakdancer
8	2nd - 29.5%	Michael Collings	Singer/Guitarist
9	1st - 36.0%	Razy Gogonea	Body-Popping Breakdancer

Using the binomial distribution with $n = 5$ and $p = \frac{1}{8}$, and with assumptions 1 to 4, what is the probability that, in a BGT semi-final with 5 shows each with 8 contestants, the last person to perform will win in only 2 shows?



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TASK E

In BGT Series 6. There were 5 semi-finals in Series 6. The number of contestants in each show was increased to 9.

Semi-final 1 (6 May)

- Guest performers: [Tulisa Contostavlos](#) - "Young"

Artist	Order	Act
Zipparah Tafari	1	Rapper
The Jive Aces	2	Jazz band
Lauren Thalia	3	Singer/guitarist
United We Stand	4	Dance troupe
Analiza Ching	5	Violinist
The Mend ²	6	Boyband
Rachel Knowland	7	Singer
Ashleigh & Pudsey	8	Dancing dog act
Only Boys Aloud	9	Male choir

Semi-final 3 (8 May)

- Guest performer: [LMFAO](#) - Medley of "Party Rock Anthem" and "Sexy and I K"

Artist	Order	Act
The Zimmers	1	Variety act
Area 51	2	Pyrotechnic dance troupe
Ashley Elliot	3	Xylophonist
Molly Rainford	4	Singer
Lucky	5	Contortionist
Loveable Rogues	6	Acoustic group
Honey Shazad	7	Singer
Twist and Pulse Dance Company	8	Dance troupe
Dennis Egel	9	Singer/entertainer

Semi-final 2 (7 May)

- Guest performer: [The Wanted](#) - "Chasing the Sun"

Artist	Order	Act
Cascade	1	Action-stunt team
Paige Turley	2	Singer
Karizma Krew	3	Dance troupe
The Show Bears	4	Performers
Fish on Percussion	5	Percussionist
Graham Blackledge	6	Organist/singer
Kai and Natalia	7	Ballroom dancers
Four Corners	8	Dance troupe
Jonathan & Charlotte	9	Opera duo

Semi-final 4 (9 May)

- Guest performer: [Labrinth](#) - "Express Yourself"

Artist	Order	Act
Chica Latina	1	Singer
Brynolf and Ljung	2	Magicians
Malakai Paul	3	Singer
Gatis Kandis	4	Comedian
The Sugar Dandies	5	Ballroom dancers
Nu Xcool	6	Dance troupe
Beatrix von Bourbon	7	Burlesque dancer
Sam Kelly	8	Singer/guitarist
Be Minor	9	Girl group

Semi-final 5 (10 May)

- Guest performer: [Rebecca Ferguson](#) - "Teach Me How to Be Loved"

Artist	Order	Act
Face Team	1	Basketball stunt team
Greig Stewart	2	Scientific laser harpist
Billy George	3	Cyr wheel acrobat
Martyn Crofts	4	Dalek impersonator
Callum Oakley	5	Comedian
Hope Murphy	6	Singer
Strictly Wheels	7	Ballroom dancers
Aquabatique	8	Synchronised swimmers
Ryan O'Shaughnessy	9	Singer-songwriter/guitarist

Set up a new binomial model stating your assumptions and calculate the probability that, in a BGT semi-final with 5 shows each with 9 contestants, the last person to perform will win 3 shows?



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TASK F

Does the order of the contestants' performance in BGT affect the probability of them winning?

Give reasons for your answer.

TASK G

Looking back on your results and using the binomial distribution:

$$X \sim \text{Bin}\left(5, \frac{1}{8}\right),$$

where X is the number of times the last act to perform wins in 5 BGT semi-finals

calculate:

$$P(X=0)$$

$$P(X=1)$$

$$P(X=2)$$

$$P(X=3)$$

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Looking at your results for the above probabilities at what point would you think that something unusual is happening?